Closing Tues: 6.3, 6.4
Closing Thur: 6.5

	Ordinary (Payments at END of each period)	**Due** (Payments at BEGINNING of each period)
FV (Balance Growing)	$F = R\frac{(1+i)^n - 1}{i}$	$F = R\frac{(1+i)^n - 1}{i}(1+i)$
PV (Balance Shrinking)	$P = R\frac{1-(1+i)^{-n}}{i}$	$P = R\frac{1-(1+i)^{-n}}{i}(1+i)$

R **= amount of each regular payment**
r = decimal interest rate
m = num. of compoundings in a year

Compute:
$i = \frac{r}{m}$ = rate at each compounding
$n = mt$ = total payments

Interest and Annuities: Since
R = payment amount, and
n = total number of payments
We have $R \cdot n$ = total amount paid.

Thus, for FV questions

Total interest earned = $F - R \cdot n$

and for PV questions

Total interest paid = $R \cdot n - P$

HW 6.3 / 6: Grandparents plan to open an account on their grandchild's birthday and contribute each month until she goes to college. How much must they contribute at the beginning of each month in an investment that pays 6%, compounded monthly, if they want the balance to be $200,000 at the end of 18 years?

And how much total interest do they earn?

"...CONTRIBUTE EACH MONTH..." ⇒ ANNUITY

"...BEGINNING..." ⇒ DUE

"...balance to be $200,000..." ⇒ FV

$$F = R \frac{(1+i)^n - 1}{i}(1+i)$$

$i = \frac{0.06}{12} = 0.005$, $n = mt = 12 \cdot 18 = 216$ MONTHLY PAYMENTS!

$F = \$200{,}000$, $R = ??$

$$200000 = R \frac{(1.005)^{216} - 1}{0.005}(1.005)$$

$$200000 = R \cdot 389.28996$$

$$R = \frac{200000}{389.28996} \approx 513.7586$$

MONTHLY PAYMENT: **$513.76**

TOTAL PAYMENTS $= R \cdot n$
$= 513.76 \cdot 216$
$= \$110{,}971.27$

INTEREST $= 20000 - 110{,}971.27$
$=$ **$89028.73**

HW Question 6.3 Problem 7(a)(b):

A small business owner contributes \$2000 at the end of each quarter to a retirement account that earns 10% compounded quarterly.

(a) How long will it be until the account is worth \$150,000? (Round your answer to the nearest *quarter*.)

ANNUITY!

"END" ⇒ ORDINARY

"UNTIL ACCOUNT IS WORTH..." ⇒ FV

$i = \frac{0.10}{4} = 0.025$

$n = mt = ?$ QUARTERS

$$F = R\,\frac{(1+i)^n - 1}{i}$$

$$150000 = 2000\,\frac{(1.025)^n - 1}{0.025}$$

FIND n!

$$75 = \frac{(1.025)^n - 1}{0.025}$$

$$1.875 = (1.025)^n - 1$$

$$2.875 = (1.025)^n$$

$$\ln(2.875) = n \ln(1.025)$$

$$n = \frac{\ln(2.875)}{\ln(1.025)} \approx 42.76796$$

$n \approx 43$ QUARTERS ⇔ $4t = 43$

⇒ $t \approx 10.69$ years

(b) Suppose when the account reaches \$150,000, the business owner increases the contributions to \$4000 at the end of each quarter. What will the total value of the account be after 15 more years?

TWO QUESTIONS!

① \$150000 LUMP SUM WILL EARN INTEREST FOR 15 YEARS

$$F = P\left(1+\frac{r}{m}\right)^{mt} = 150000\left(1+\frac{0.1}{4}\right)^{4(15)} \approx \$659{,}968.46$$

② \$4000 END OF EACH QUARTER FV, ordinary annuity!

$$F = R\,\frac{(1+i)^n - 1}{i} = 4000\,\frac{(1.025)^{60} - 1}{0.025} \approx \$543{,}966.36$$

TOTAL MONEY IN 15 YRS $= 659{,}968.46 + 543{,}966.36$

$= \$1{,}203{,}934.82$

(Old Final Question)

Immediately after graduating from college, Julian started a savings plan, depositing $200 at the end of each month in an account paying 3% interest, compounded monthly. After 6 years, Julian stopped making payments but left the money in the account, gaining interest at the same rate, for an additional 4 years.

(a) How much money is in the account in 6 years?

(b) How much money is in the account in 10 years?

(a) FIRST SIX YEARS, ANNUITY, FV, ORDINARY

$$F = R\,\frac{(1+i)^n - 1}{i} \qquad i = \frac{0.03}{12} = 0.0025$$

$$n = 12 \cdot t = 72 \text{ months}$$

$$F = 200\,\frac{(1.0025)^{72} - 1}{0.0025}$$

$$F \approx 15755.8774 \qquad \boxed{\$15{,}755.88}$$

(b) NEXT FOUR YEARS, LUMP SUM!

$$F = P\left(1 + \frac{r}{m}\right)^{mt} \qquad i = \frac{r}{m} = 0.0025$$

$$n = mt = 12 \cdot 4 = 48 \text{ months}$$

$$F = 15755.88\,(1.0025)^{48}$$

$$\approx \boxed{\$17{,}762.04}$$